Ratios & Proportions

Rates/Ratios/Proportions

An example of a rate is 25 miles/hr, which means the object will move 25 miles for every hour that passes.

An example of a ratio is 2 eggs:3 ounces of flour, which means the recipe calls for 2 eggs for every 3 ounces of flour used.

A statement that 2 rates or ratios are equal is a proportion.

$$\frac{2\ eggs}{3\ oz.flour} = \frac{8\ eggs}{12\ oz.flour}$$

Check by cross multiplying.

$$\frac{2}{3} = \frac{8}{12}$$

$$24 = 24$$

Setting Up a Proportion

When setting up a proportion, always write down how you will set it up

$$\frac{pages}{minute}$$

Then insert the values.

$$\frac{pages}{minute}\ \frac{13}{4} = \frac{x}{22}$$

Use cross multiplication to solve for a missing value in a proportion

$$\frac{13}{4} = \frac{x}{22}$$

$$286 = 4x$$

$$\frac{286}{4} = \frac{\cancel{4}x}{\cancel{4}}$$

$$71.5 = x$$

Practice 1

The ratio of boys to girls in a class is 3: 2. How many boys are there if there are 12 girls?

Problems with Ratios and a Total

To solve for values when a ratio and a total is given, multiple each number in the ratio by *x* and then create an equation in which they all add up to the total. Then solve for *x*, and use that value of *x* to determine the other values.

Example:

The ratio of boys to girls in a class is 2:3. If there are 25 students, how many boys are there?

Boys + Girls = 25

$$2x + 3x = 25$$

$$5x = 25$$

$$\frac{5x}{5} = \frac{25}{5}$$

$$x = 5$$

Now, we can find the number of boys

$$\text{Boys} = 2x = 2(5) = \boxed{10}$$

Practice 2

A recipe for cake calls for 2.5 cups of flour to be mixed with 2 cups of sugar. The two ingredients should be mixed first before other ingredients are added. If the total mixture of flour and sugar should be 18 cups, how many cups of sugar should be used?

Use Matching Units in the Proportion

Make sure to correct the units if they are different, before setting up the proportion.

Example

Tammy can read 10 pages in 4 minutes. How many pages can she read in 2 hours?

$$\frac{\text{Pages}}{\text{min}} \quad \frac{10}{4} = \frac{x}{2}$$

Not the correct units

2 hours = 120 minutes

$$\frac{10}{4} = \frac{x}{120}$$

Now, it has the correct units

$$1200 = 4x$$

$$\frac{1200}{4} = \frac{\cancel{4}x}{\cancel{4}}$$

$$\boxed{300} = x$$

Practice 3

Michelle works in a shirt printing company. If she can print 40 shirts per hour and works 6 hours each day, how many shirts can she print if she works three days?

Ratios Involving Formulas

When given a formula, we can determine how one value will be affected by altering other values.

For example,

The weight of an object, F_g, is found by multiplying the mass of an object, m, by gravity, g.

$$F_g = mg$$

If the mass, m, is doubled, the weight, F_g, will be doubled.

Just like when we solve equations, whatever we do to one side of the equation, we have to do to the other side of the equation.

SUPER TIP
To see how doubling the value of an input variable in a formula will impact the result, insert a 2 for the variable and 1s for the other variables. Then, compare it to what the value would have been if all 1s were used.

For example, to see how double the mass would affect the weight:

$$F_g = mg = (2)(1) = 2$$

Double

$$F_g = mg = (1)(1) = 1$$

We end up with 2, which is twice as big as 1, so it means the new weight will be twice as big (doubled).

This is very useful for more complicated formulas, including ones with variables that are raised to an exponent.

Example

The formula to calculate kinetic energy is $K = \frac{1}{2}mv^2$. How will K be affected if we double v?

$$K = \frac{1}{2}mv^2$$
$$= \frac{1}{2}(1)(2)^2$$
$$= \frac{1}{2}(4)$$
$$= 2$$

$$K = \frac{1}{2}mv^2$$
$$= \frac{1}{2}(1)(1)^2$$
$$= \frac{1}{2}$$

Quadruple

2 is four times as big as $\frac{1}{2}$, so doubling v quadruples K.

Practice 4

$$F_g = \frac{Gm_1m_2}{r^2}$$

The formula above is used to calculate the gravitational attraction between two masses. What will happen to the value of F_g if m_1 is tripled and r is tripled?

A) F_g will be tripled.
B) F_g will be nine times as large as its original value.
C) F_g will remain the same.
D) F_g will be one-third of its original value

Proportions in Shapes

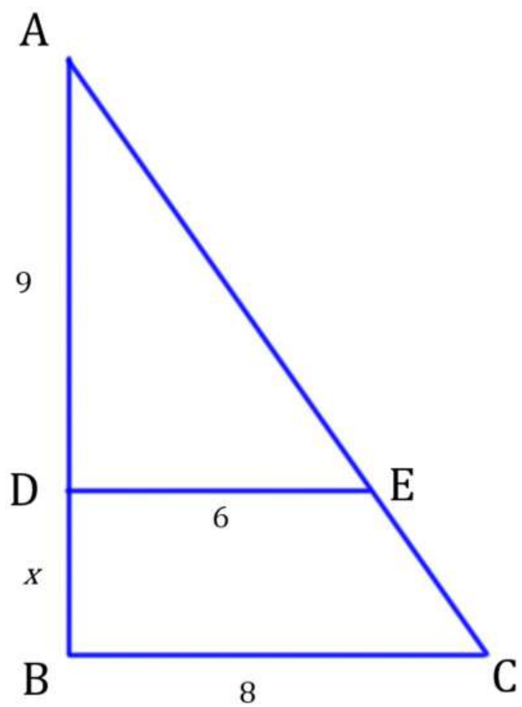
Similar figures and shapes have proportionate side lengths.

A useful way to write down how we are setting up a proportion regarding geometric shapes is:

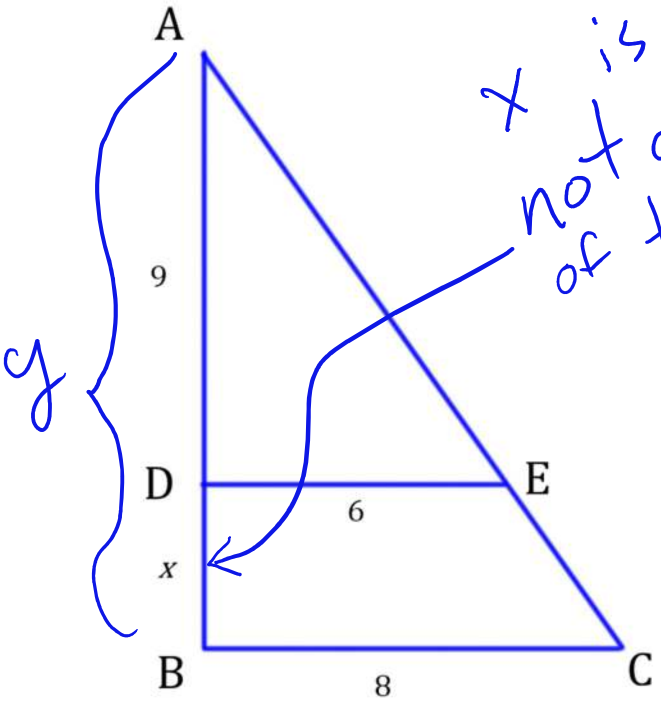
$$\frac{\text{Large}}{\text{Small}}$$

SUPER TIP
When dealing with proportionate shapes, make sure that you're dealing with corresponding lengths. It is common for a question to ask for the length of a segment and not a full side.

Example



What is the value of x in the figure above?



x is not a side of the Δ

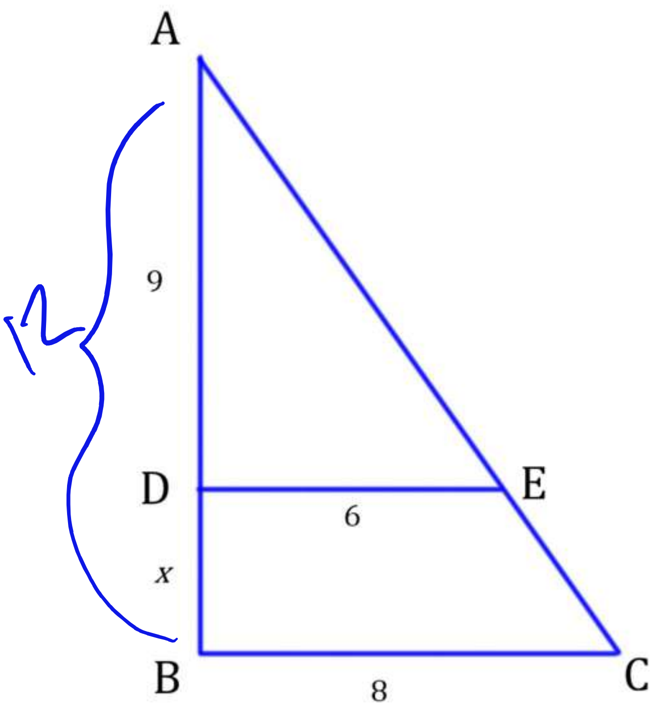
$$\frac{\text{BIG}}{\text{SMALL}}$$

$$\frac{y}{9} = \frac{8}{6}$$

$$6y = 72$$

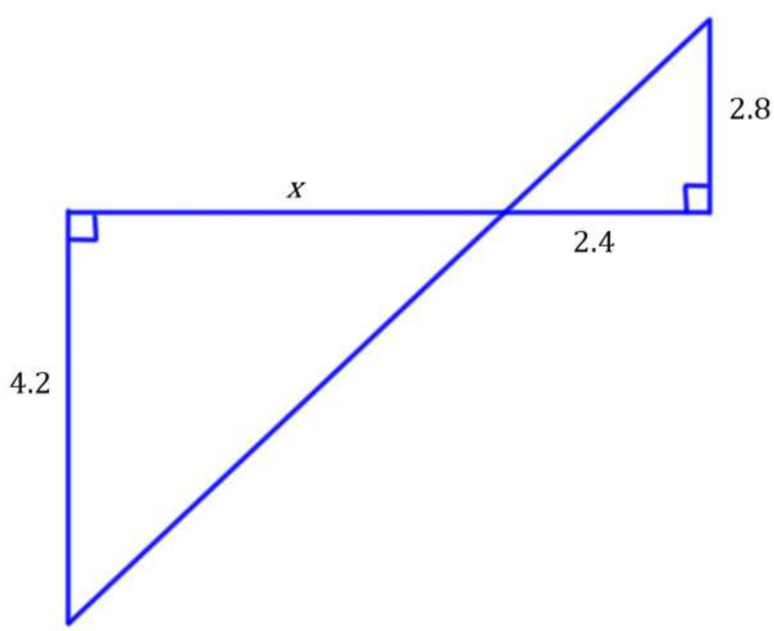
$$\frac{6y}{6} = \frac{72}{6}$$

$$y = 12$$



$$x + 9 = 12$$
$$-9 \quad -9$$
$$x = \boxed{3}$$

Practice 5



What is the vale of x in the figure above?

More Practice

Practice 6

The ratio of chocolate chip cookies to oatmeal cookies in a display case is 7:2. If there are 56 chocolate chip cookies, how many oatmeal cookies are there?

Practice 7

To prepare concrete it is advised to use 600 pounds of cement, 1450 pounds of sand, 1600 pounds of stone and 260 pounds of water. If it is estimated that a project will require 46,920 pounds of concrete. How many pounds of water will be needed?

Practice 8

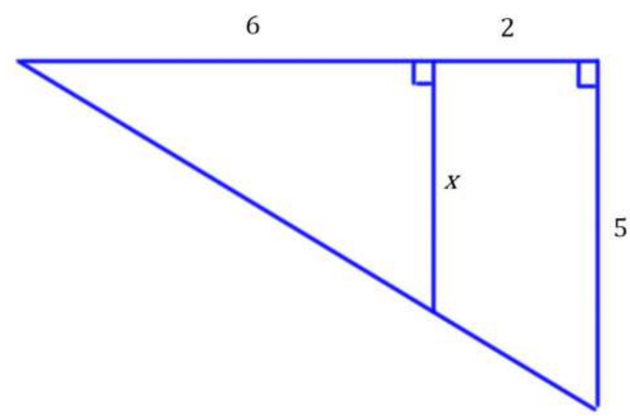
Noriko can type 50 words per minute. How many words can she type in 3 hours?

Practice 9

The formula $P = \frac{Fd}{t}$ is used to calculate power, P. If t is doubled and F is halved, how will that affect P?

A) P will be doubled.
B) P will be halved.
C) P will be quartered.
D) P will be quadrupled.

Practice 10



What is the value of x in the figure above?

Answers to Practice Problems

Practice 1: 18

Boys : Girls

3 : 2

Girls = 12

$$\frac{B}{G} \quad \frac{3}{2} = \frac{x}{12}$$

$$36 = 2x$$

$$\frac{36}{2} = \frac{2x}{2}$$

$$18 = x$$

Practice 2: 8

Flour : S

2.5 : 2

18 total

Flour + Sugar = 18

$$2.5x + 2x = 18$$

$$4.5x = 18$$

$$\frac{4.5x}{4.5} = \frac{18}{4.5}$$

$$x = 4$$

Sugar = 2x

= 2(4)

= 8 cups

Practice 3: 120

40 shirts/hr

6 hrs/day

3 days

$$\frac{sh}{h} \quad \frac{40}{6} = \frac{x}{?}$$

Need total hours

$$3 \text{ days} \times 6 \frac{\text{hrs}}{\text{day}} = 18 \text{hrs.}$$

$$\frac{40}{6} = \frac{x}{18}$$

$$720 = 6x$$

$$\frac{720}{6} = \frac{\cancel{6}x}{\cancel{6}}$$

$$120 = x$$

Practice 4: D

$$F = \frac{G m_1 m_2}{r^2}$$

triple m_1

triple r

$$F = \frac{(1)(3)(1)}{(3)^2}$$

$$F = \frac{3}{9} = \frac{1}{3}$$

Original:

$$F = \frac{G m_1 m_2}{r^2}$$

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$$F = \frac{(1)(1)(1)}{(1)^2}$$

$$F = 1$$

$$1 \rightarrow \frac{1}{3}$$

1/3 of original

Practice 5: 3.6

$$\frac{\text{BIG}}{\text{SMALL}} \quad \frac{4.2}{2.8} = \frac{x}{2.4}$$

$$10.08 = 2.8x$$

$$\frac{10.08}{2.8} = \frac{\cancel{2.8}x}{\cancel{2.8}}$$

$$3.6 = x$$

Practice 6: 16

CH : OAT

7 : 2

CH = 56

OAT = ?

$$\frac{\text{CH}}{\text{OAT}} \quad \frac{7}{2} = \frac{56}{x}$$

$$7x = 112$$

$$\frac{\cancel{7}x}{\cancel{7}} = \frac{112}{7}$$

$$x = 16$$

Practice 7: 3,120

Cement = 600

Sand = 1450

stone = 1600

Water = 260

Total 46920

$C+S+St+W=46920$

$600x+1450x+1600x+260x=46920$

$3910x=46920$

$\frac{3910x}{3910}=\frac{46920}{3910}$

$x=12$

Water $=260x$

$260(12)$

$\boxed{3120}$

Practice 8: 9,000

50 words/min

words in 3hrs = ?

3hrs = 180 min

$\frac{w}{min} \quad \frac{50}{1}=\frac{x}{180}$

$9{,}000=x$

Practice 9: C

$P=\frac{Fd}{t}$

double t

halve F

P = ?

$$P = \frac{Fd}{t}$$

$$P = \frac{\left(\frac{1}{2}\right)(1)}{2^2}$$

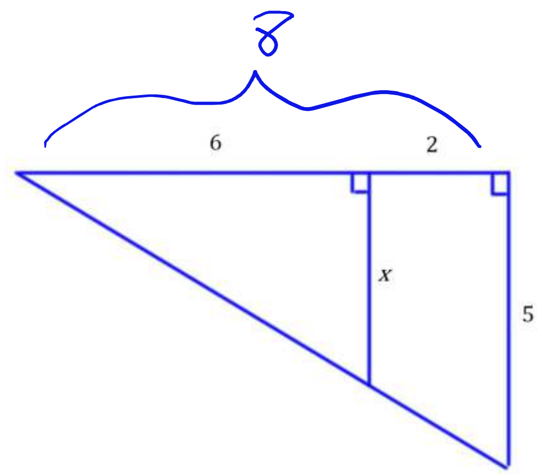
$$P = 1/4$$

Original:

$$P = \frac{Fd}{t} = \frac{(1)(1)}{(1)} = 1$$

P is 1/4 of the original

Practice 10: 3.75



$$\frac{\text{BIG}}{\text{SMALL}} \quad \frac{8}{6} = \frac{5}{x}$$

$$8x = 30$$

$$\frac{8x}{8} = \frac{30}{8}$$

$$x = 3.75$$

Data & Probability

Surveys

It is common to see a question on the SAT that asks why a survey conducted is unreliable. Most of the times it is unreliable because of a possible bias related to who was surveyed, where they were surveyed, when they were surveyed, or how they were surveyed.

When choosing the group of people to be surveyed, the group should be a random sample of the entire population to be surveyed. For example, a survey to determine a town's favorite pizzeria should randomly survey people from the town.

Most of the times, the answer won't be because of the sample size.

Here are some examples of creating a bias based on who was surveyed:

1. A survey of the favorite class of students at a high school would not be reliable if only students from the freshmen class were surveyed or if any of the grades were excluded from the survey.
2. A survey of car enthusiasts to find out which car is the most popular would not be reliable if the survey was sent to a mailing list of people who earned a certain type of sports car.

Here are some examples of creating a bias based on location:

1. A survey of adults to determine the average number of children per household would not be reliable if the survey was done at a location where there are likely children because most of the adults there will have children. For example, at a playground the adults there are likely there with children, so it is less likely to survey an adult with no children.
2. A survey of people's favorite activity would not be reliable if it was held at a place related to one of those activities. For example, at a movie theater the people are likely to choose going to the movies or at a bookstore the people are more likely to choose something related to reading.
3. A survey of whether people prefer watching sports in person or at home would not have reliable results if the survey was done outside a stadium before a game. The people there may be going to the game to watch it in person. Plus, it excludes the people who are home and waiting for the game to start.

Here are some examples of creating a bias based on when they were surveyed:

1. A survey of the most popular occupation in a town would not be reliable if people were randomly survey inside a supermarket on a weekday during the workday because people with professions that require them to work during normal work hours would be excluded from the survey.
2. A survey done to determine the opinion of the population of a town will likely not be reliable if done at a time when the majority of the population is expected to be sleeping.

An example of how a bias could be created is:

A survey that requires people to use a specific type of phone or through a specific social media application because it will exclude people who don't have the specific type of phone of the specific application.

However, if the purpose is to survey owners of a specific phone, then it should be required that they use that type of phone to participate.

SUPER TIP
If a question asks why a reasonable conclusion is not likely, it won't be because of the size of the group surveyed or because people were allowed to opt out of answering

Practice 1

For her class project, Chloe wanted to determine the percent of students who thought the school should spend part of its budget on renovating the sports field. One Saturday, she interviewed every fourth student who entered the stands to watch the high school's football team. If someone didn't want to participate, she simply allowed them to not participate. When she was done, she had interviewed 40 students and 5 students refused to participate. Which of the following factors makes it least likely that a reliable conclusion can be drawn about the preferences of all of the students of Chloe's school?

A) The sample size
B) The number of people who refused to participate
C) Where the survey was given
D) The fact that Chloe interviewed every fourth student instead of every student who entered

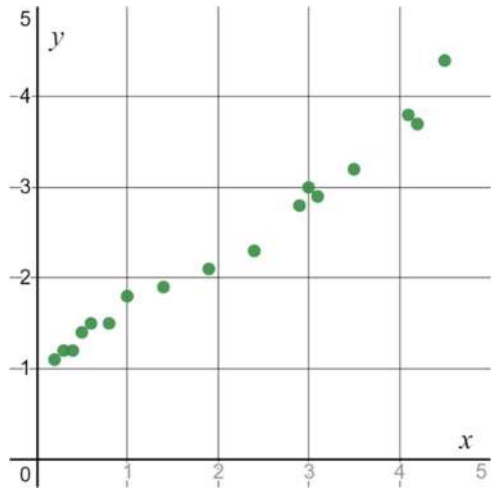
Scatterplots

Correlation

When a relationship appears to exist among data points appear it is said that there is correlation.

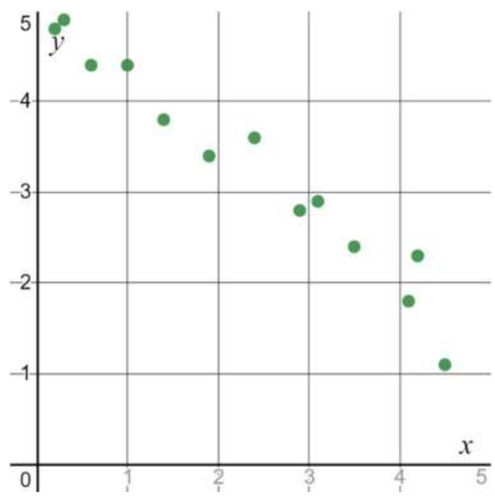
For example, as time increases, the cost of college increases. Since they both go up, it is said that there is positive correlation.

Below is an example of a positive correlation between x and y.



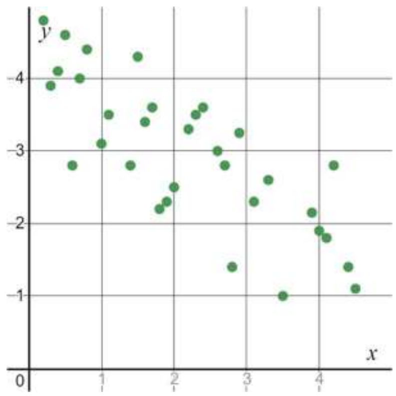
Another example would be as time increases the population of a specific species decreases. Since one value increases while the other decreases, it is said that there is a negative correlation.

Below is an example of a negative correlation between x and y.

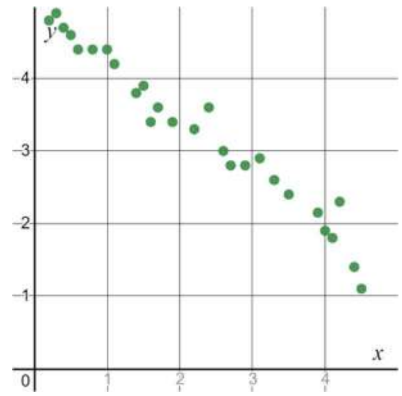


The more apparent that there is a relationship, the stronger the correlation is said to be.

Below are two examples of a negative correlation between x and y. However, the graph on the right shows a stronger correlation than the graph on the left.



Negative correlation

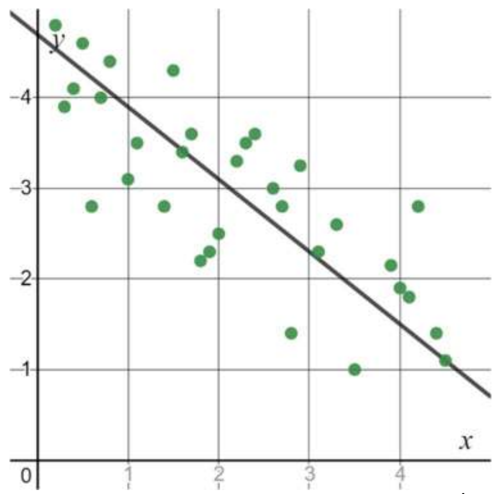


Strong negative correlation

Trendlines

Trendlines (lines of best fit) can be applied to scatterplots that show correlation.

For example, a trendline has been added to the scatterplot below.



The line of best fit is modeled by $f(x) = -\frac{4}{5}x + 4.7$

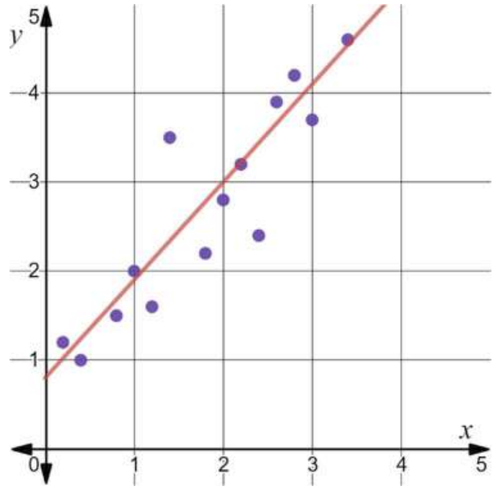
Obviously, each point on the scatterplot is not on the line of best fit. However, it does show the trend of the data and can be used to make estimations.

Based on the line of best fit, we can estimate the y-value when x is 4 by substituting 4 into the function $f(x) == \frac{4}{5}x + 4.7$.

$$y = -\frac{4}{5}x + 4.8$$
$$y = -\frac{4}{5}(4) + 4.8$$
$$y = 1.6$$

If we look at the graph, we can see that when x is 4, y is approximately 1.9, not 1.6. It's important to remember that we use the line of best, or a trendline, to estimate values.

Practice 2



The scatterplot above shows the average frequency of homes, y, for each home value in millions of dollars, x. The frequency of homes, $f(x)$, can be estimated with the function $f(x) = 1.1x + 0.8$. Based on the information in the graph above, what is the estimated frequency of homes with a value of $3 million?

A) 2
B) 3.8
C) 4.1
D) $2{,}000{,}000$

Mean, Median, Mode, & Range

Mean (Average)

$$Mean = \frac{Sum\ of\ Items}{Number\ of\ Items}$$

For example, let's find the mean of: 80, 90, 90, 95, 95

$$Mean = \frac{80 + 90 + 90 + 95 + 95}{5} = 90$$

Many times, on the SAT it is important to be able to find the sum of the items in order to solve the problem.

$$Sum\ of\ Items = (Average) * (Number\ of\ Items)$$

Example

Mike's average for 5 math tests is 90. His teacher is going to drop his lowest test, a 70. What will his average be for the remaining four tests?

It is important to realize that we do not need to know what the other four test grades are. We only need to know what they add up to.

First let's find his total including all 5 tests

$$Sum\ of\ Items = (Average) * (Number\ of\ Items)$$
$$Sum\ of\ Items = (90) * (5)$$
$$Sum\ of\ Items = 450$$

Now to find his new total, after dropping the 70, we simply have to subtract 70.

$$New\ Sum\ of\ Items = 450 - 70$$
$$New\ Sum\ of\ Items = 380$$

To find his new average we use the Arithmetic Mean (average) formula

$$Arithmetic\ Mean = \frac{Sum\ of\ Items}{Number\ of\ Items}$$

$$Arithmetic\ Mean = \frac{380}{4}$$

$$Arithmetic\ Mean = 95$$

Practice 3

Jose's average after 5 tests is 75. If his grades for the first 4 tests are 90, 80, 62 and 71, what was his grade for his 5th test?

Practice 4

Mark and Sandra collected seashells from the beach. They each collected four seashells. The mean of the masses of the rocks that Sandra collected is 1 gram more than the mean of the masses of seashells that Mark collected. What is the value of x?

	Masses (grams)			
Mark	12.4	10.2	12.0	14.2
Sandra	13.0	10.6	11.7	x

Median, Mode, & Range

Median: The middle number in a set of numbers when arranged in order numerically.
Mode: The number that appears most often. It is possibly to have more than one mode.
Range: The difference between the largest number and the smallest number.

Example

Mary had the following bowling scores: 125, 215, 136, 195, and 202. How much greater is the mean score than the median?

125, 215, 136, 195, 202

To find the median, put the scores in order:

125, 136, 195, 202, 215

median = 195

$$\text{mean} = \frac{125+136+195+202+215}{5}$$

$$= \frac{873}{5} = 174.6$$

$$195 - 174.6 = \boxed{20.4}$$

Mean, Median, Mode and Range of Data from a Table

Mean from a Table

Tests Scores	# of students
100	1
90	3
80	5
70	0
60	1
50	0

To find the mean from a table, total all of the rows and then divide by the total frequency of values.

We can find the total of the values by multiplying each value by the frequency then adding.

In the table above, there is one 100, three 90s, five 80s, etc. Instead of adding them individually, we can find the total of each row.

Tests Scores	# of students	
100	1	100
90	3	270
80	5	400
70	0	0
60	1	60
50	0	0

Then we can add up those totals to get 830, and we can find the total number of values by adding up the frequency.

Tests Scores	# of students	
100	1	100
90	3	270
80	5	400
70	0	0
60	1	60
50	0	0
	10	830

Finally, we can find the mean (average)

$$\frac{830}{10} = 83$$

Practice 5

explanation not in student copy

Tests Scores	# of students	
100	3	300
90	7	630
80	4	320
70	6	420
	20	1670

$$\frac{1670}{20} = \boxed{83.5}$$

What is the mean test score in the table above?

Median from a Table

To find the median, we don't want to write out all of the values. Instead, we want to determine which term will be the middle term from the table.

To find which term will be the median, use the following formula

$$Median\ Term = \frac{n+1}{2}$$

Where *n* is the number of terms. Note: If there is an even number of terms, we will get an answer that ends in ".5". This means the median is the average of the numbers in the places before and after.

Example

If there are 13 numbers, which one will be the median?

$$\frac{13+1}{2} = \frac{14}{2} = 7$$

7^{th} term

Example

If there are 14 terms, which one will be the median?

$$\frac{14+1}{2} = \frac{15}{2} = 7.5$$

Avg of the 7^{th} & 8^{th} terms

Example

Tests Scores	# of students
100	15
90	30
80	50
70	10
60	5
50	20

Based on the data in the table above, what is the median test score?

First let's find out how many terms there are, then what term the median is.

Tests Scores	# of students
100	15
90	30
80	50
70	10
60	5
50	+ 20

130 terms

$$\frac{130+1}{2} = 65.5$$

Avg of 65^{th} + 66^{th} terms

Now, we have to find the 65^{th} and 66^{th} terms. Let's start counting from the top.

Tests Scores	# of students	
100	15	15
90	30	45
80	50	95
70	10	
60	5	
50	20	

$\leftarrow$ 80 is the 46^{th} through 95^{th} term, so it is also the 65^{th} and 66^{th}

Let's also count from the bottom to be certain.

Tests Scores	# of students
100	15
90	30
80	50
70	10
60	5
50	20

85 ←
35
25
20

80 is the 36^{th} through 85^{th}, so it is the 65^{th} and 66^{th}

The median in 80, because the average of 80 and 80 is 80.

explanation not in student copy

Practice 6

Tests Scores	# of students
100	3
90	5
80	2
70	9

3
8
10
19

$3+5+2+9=19$

$$\frac{n+1}{2} = \frac{19+1}{2} = 10$$

10^{th} term

What is the median test score in the table above?

10^{th} term is $\boxed{80}$

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Mode from a Table

Tests Scores	# of students
100	1
90	3
80	5
70	0
60	1
50	0

Mode is the easiest to find from a table, but still trips up some students.

The value with the greatest frequency is the mode, so in this case 80 is the mode because 5 students scored an 80 (the frequency is 5).

Some student might get tripped up and pick 0 or 1 because they appear the most, twice.

However, we have to realize that the second column represents frequencies.

Really, this is the data in the table:
100, 90, 90, 90, 80, 80, 80, 80, 80, 60

Now, it is clearer that the mode is 80.

explanation not in student copy

Practice 7

Tests Scores	# of students
100	3
90	7
80	4
70	6

What is the mode test score in the table above?

most frequent : $\boxed{90}$

Range from a Table

Tests Scores	# of students
100	1
90	3
80	5
70	0
60	1
50	0

Finding the range from a table is fairly straight forward, simply find the difference between the highest value and the lowest.

Be certain to subtract values and not frequencies.

Also, make sure the value exists in the data set. For example, if we look at the table above, it looks like 50 is the lowest value, so the range would be $100 - 50 = 50$. However, there are no 50s (the frequency is 0), so the lowest test score is actually 60.

The range is $100 - 60 = 40$.

Practice 8

The table below shows the number of houses that sold for different price values in the town of North Fakeness last month. How much greater was the mode sale price than the median sale price?

Sale Price ($)	**Frequency**
350,000	6
400,000	3
450,000	7
500,000	8
550,000	12

Drawing Appropriate Conclusions

One of the most important things to remember about drawing conclusions from a survey is that we can only infer what it likely true about the entire population of that group.

For example, if a survey of 50 8th graders from a school in New York City reveals that 40% of the students surveyed like football more than baseball, we can infer that it is likely that 40% of the entire 8th grade class in that school likes football more than baseball. However, it would be in correct to apply that information to the entire school since only 8th graders were surveyed, and it would be wrong to apply that information to all 8th graders in New York City because only 8th graders from one school were surveyed.

Practice 9

In a small town there are two types of owls, A-owls and B-owls. A-owls have a slightly bigger beak but a smaller wingspan. To study the owl population of the small town, a researcher randomly caught 50 owls, studied them quickly, and then released them back into the wild. 35 out of the 50 owls were A-owls. Based on the random sample, which of the following conclusions can appropriately be made?

A) If there are 5,000 owls in the small town, then 3,500 of them will likely be A-owls.
B) If there are 5,000 owls in the small town, then 3,500 of them will likely be B-owls.
C) There are more A-owls that B-owls in the USA.
D) Bird with bigger beaks are more likely to be caught by a researcher than birds with smaller beaks.

Filling in Tables

Sometimes data will be presented in a table. In those tables one column and/or one row may represent totals.

The values should add up correctly and give the right totals.

For example, let's look at the following table

	Saw the Movie	Didn't See the Movie	Total
Men	58	192	250
Women	238	12	250
Total	296	204	500

	Saw the Movie	Didn't See the Movie	Total
Men	58	192	250
Women	+ 238	+ 12	+ 250
Total	296	204	500

The total number of people who saw the movie (296) equals the sum of number of men who saw it (58) and the number of women who saw it (238).

The total number of people who didn't see the movie (204) equals the sum of number of men who didn't see it (192) and the number of women who didn't see it (12).

The total number of people (500) is the sum of the total number of men (250) and the total number of women (250).

	Saw the Movie	Didn't See the Movie	Total
Men	58 +	192 =	250
Women	238 +	12 =	250
Total	296 +	204 =	500

The total number of men (250) equals the sum of the number of men who saw the movie (58) and the number of men who didn't see the movie (192).

The total number of women (250) equals the sum of the number of women who saw the movie (238) and the number of women who didn't see the movie (204).

The total number of people (500) equals the sum of the number people who saw the movie (296) and the sum of the number of people who didn't see the movie (204).

Since we can add up the values, we can solve for missing values.

Practice 10

	Male	Female	Total
Arabian	45		94
Clydesdale		72	
Total			216

The table above shows the type of and gender of the horses at specific farm in North Carolina. Based on the data in the table above, how many horses are male Clydesdales?

Probability

Probability is the likeliness of something occurring.

For example, if you roll a die, there is a $1/6$ chance of it landing on 3.

$$Probabilty = \frac{\#\ of\ favorable\ outcomes}{total\ possible\ outcomes}$$

For rolling a 3, there is only one 3 on a die, so the number of favorable outcomes is 1. There are 6 possible outcomes, so the number of total possible outcomes is 6. Therefore, the probability is $1/6$.

Probability can be expressed as a fraction, a decimal or a percent.

$$\frac{1}{6} = 0.167 = 16.7\%$$

SUPER TIP
Many times on the SAT, probability questions will be based on a table of data. It is very important to read the question carefully to identify the total possible outcomes (out of which group we are finding the probability of something occurring).

Example

Television Programs Watched Each Week

	None	**1 to 3**	**4 or more**	**Total**
Group A	5	38	57	100
Group B	7	45	48	100
Total	12	83	105	200

For a science project, a student interviewed 200 people. Group A consisted of 100 adults and Group B consisted of 100 high school students.

Part 1
If a person is chosen at random from Group A, what is the probability that the person watches 4 or more television programs each week?

Television Programs Watched Each Week

	None	1 to 3	4 or more	Total
Group A	5	38	57	100
Group B	7	45	48	100
Total	12	83	105	200

Group A + watches 4 or more

Total of Group A

$\boxed{57/100}$

Part 2
If a person is chosen at random from those who watch at least 1 television program per week, what is the probability that the person belonged to Group B?

Television Programs Watched Each Week

	None	1 to 3	4 or more	Total
Group A	5	38	57	100
Group B	7	45	48	100
Total	12	83	105	200

Watch at least 1 television program per week + Group B: 93

Watch at least 1 television program per week: 188

$\boxed{93/188}$

Part 3
If a person who participated in the science project is chosen at random, what is the probability that the person was from Group A and watched 1 to 3 television programs per week?

Television Programs Watched Each Week

	None	1 to 3	4 or more	Total
Group A	5	38	57	100
Group B	7	45	48	100
Total	12	83	105	200

Group A & watched 1 to 3 televisision programs

participated

$$\frac{38}{200} = \boxed{\frac{19}{100}}$$

Practice 11

	6th Graders	**7th Graders**	**8th Graders**	**Total**
School A	345	395	326	1,066
School B	410	445	432	1,287
Total	755	840	758	2,353

The table above shows the number of 6th, 7th, and 8th graders at two different middle schools.

Part 1
If a student from School A is chosen at random, what is the probability that the student will be a 6th grader?

Part 2
If a 7th or 8th grader is chosen at random from either of the two schools, what is the probability that the student will be from School B?

Standard Deviation

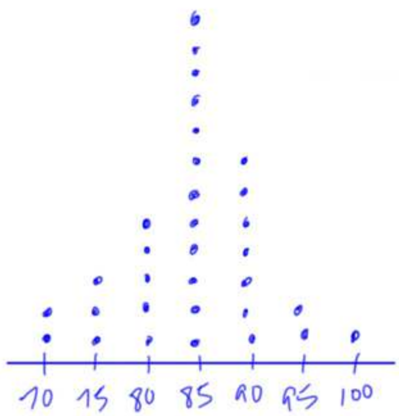
In many cases, values tend to be centrally located.

For example, the test grades of a class might be as follows:

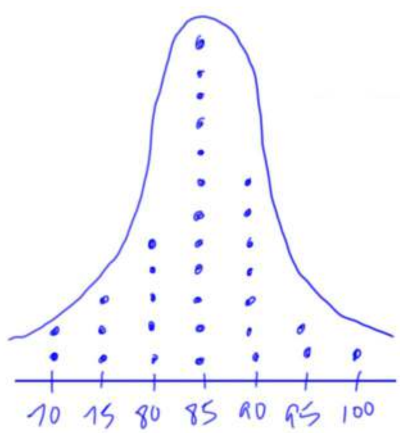
Test Grade	Frequency
70	2
75	3
80	5
85	12
90	7
95	2
100	1

The average test grade in the class is 84.5. From the table, we can see that most of the test grades are close to 84.5.

A dot plot of the data would give us the following:



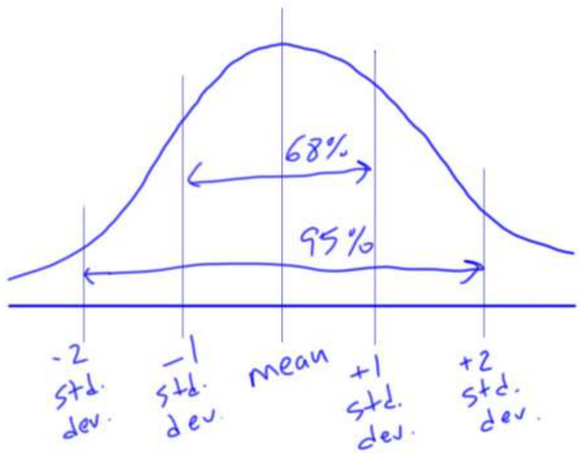
If follows a bell shape.



When data is normally distributed, we can use standard deviation to make estimates about the data. Two concepts that come up on the SAT are: about 68% of the values are estimated to be within one standard deviation of the mean and about 95% of the values are estimated to be within two standard deviation of the mean.

For example, if the mean chemistry final exam score for a school is 78 and the standard deviation is 6, we can estimate that 65% of the students scored between 72 (6 less than the mean) and 84 (6 greater than the mean. 95% of the values will be within two standard deviations (12) of the mean, so we can estimate that 95% of the students scored between 66 (12 less than the mean) and 90 (12 more than the mean).

Below is a diagram to help show the concept visually.



SUPER TIP
The closer the values are to the mean, the smaller the standard deviation.

For example, below are the grades for two classes. Both classes have 23 students, a low score of 75, and a high score of 100. However, they have different standard deviations.

Class A		Class B	
Test Score	Frequency	Test Score	Frequency
75	1	75	5
80	3	80	4
85	5	85	2
90	8	90	5
95	4	95	3
100	2	100	4

Most of the scores in Class A are between 85 and 95. But the scores in Class B are spread throughout the range. As a result, the standard deviation of Class A's scores is smaller than the standard deviation of Class B's scores.

Practice 12

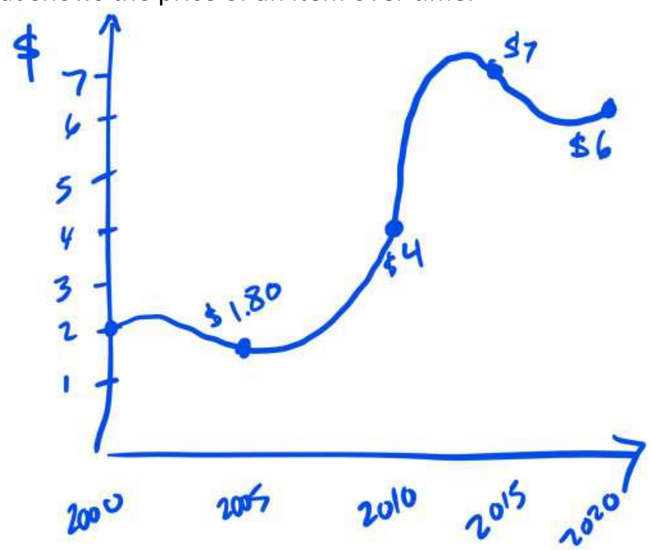
The weights of 100 volunteers who took part in a health study were recorded. The mean weight was 185 pounds, and the standard deviation was 8 pounds. Which of the following ranges or weights would it be reasonable to assume that 64 of the volunteers' weights, w, fall within?

A) $181 < w < 189$
B) $177 < w < 185$
C) $185 < w < 193$
D) $177 < w < 193$

Average Increase/Decrease

To find the average increase or decrease of a value over time, we just have to focus on the initial and final values.

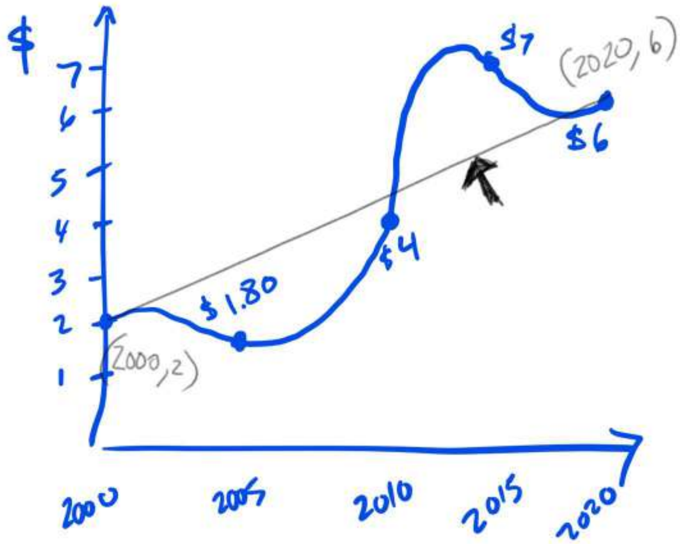
Let's look at this graph that shows the price of an item over time.



The price changed at different rates at different times. In fact, at times the price was increasing and at other times it was decreasing.

SUPER TIP
We can find the average increase/decrease by finding the average rate of change by using the slope formula. In most cases, the value changes with time, so the values will be treated as y-values and the times will be treated as x-value.

We can find the average increase or decrease by finding the slope of the line drawn from start to finish.



$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6-2}{2020-2000}$$

$$m = \frac{4}{20} = .20$$

$$\$0.20/yr$$

The average increase is $0.20/year.

We can find the average change from a table also.

Example

Year	Average Home Price ($)
1970	180,000
1980	195,000
1990	220,000
2000	250,000
2010	205,000
2020	230,000

The table above shows the average home prices in Makebelieveville. What is the average annual increase in average home prices from 1980 to 2010?

In 1980, Avg \$195,000
In 2010, Avg \$205,000

$$\frac{205{,}000-195{,}000}{2010-1980}$$

$$\frac{10{,}000}{30}$$

\$333.33/yr

Practice 13

A plant is grown in a laboratory to test the effectiveness of a new plant food. Each day, the height of the plant is recorded. The data is shown in the table below. What is the average daily increase of the plant's height, in inches per day?

Day	1	2	3	4	5	6	7
Height (in.)	2.40	2.45	2.49	2.52	2.58	2.64	2.74

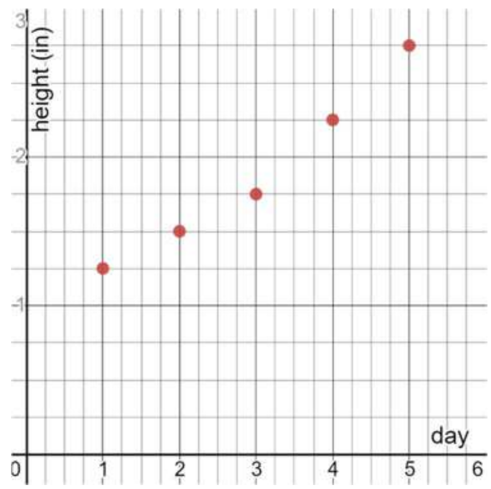
Practice 14

Two plants were grown in a different laboratory to test the effectiveness of different combinations of light, water and plant food. Each day the heights of the plants were recorded. The data from two of the plants under different condition are shown below. Which of the following statements are true?

Plant A

Day	Height
1	3.25
2	3.28
3	3.32
4	3.45
5	3.51

Plant B



A) Plan A grew 0.76 inches per day faster than Plan B
B) Plant B grew 0.44 inches per day faster than Plant A
C) Plant B grew 0.31 inches per day faster than Plant A.
D) Plant A and Plant B grew at the same rate

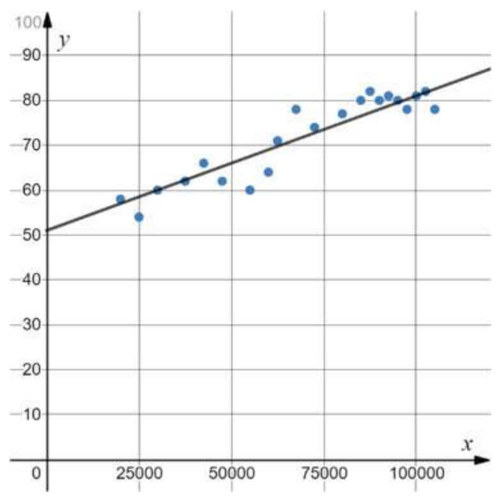
More Practice

Practice 15

In order to determine if a new weight-loss pill is successful in causing weight-loss, a research study was conducted. From a large population of people who wanted to lose weight, 500 participants were selected at random. Half of the participants were randomly assigned to receive the new weight-loss pill, and the other half did not receive the pill. The resulting data showed that participants who received the pill lost more weight than the participants who didn't. Based on the design and results of the study, which of the following is an appropriate conclusion?

A) The new weight-loss pill is better than all other weight-loss treatments.
B) The new weight-loss pill will work for anyone who takes it.
C) The new weight-loss poll is likely to help people who use it lose weight.
D) The new weight-loss poll will cause a substantial amount of weight-loss.

Practice 16



A survey was conducted that asked people to rate their happiness on a scale of 0 to 100. The average ratings for each salary amount, y, was computed and plotted along with the salary, x. The line of best fit is also shown and has the equation $y = \frac{3}{10{,}000}x + 51{,}000$. Which of the following best explains how the number $\frac{3}{10{,}000}$ in the equation relates to the scatterplot?

A) The lowest happiness rating is about $\frac{3}{10{,}000}$.
B) It can be estimated that the happiness rating increases by 3 for every additional \$10,000 in salary.
C) The happiness rating for someone with no income would be estimated to be $\frac{3}{10{,}000}$.
D) Only 3 out of every 10,000 people were surveyed.

____________∇____________

Practice questions 17, 18 and 19 refer to the following information.

	Engineering	Liberal Arts	Medical	Law	Business	Total
High School A	252	43	176	102	74	647
High School B	58	75	295	202	54	684
High School C	8	54	2	7	24	95
High School D	24	653	72	35	430	1,214
Total	342	825	545	346	582	2,640

The table above shows the preferred fields of study of high school juniors from four different high schools.

Practice 17

Based on the table, if a student who prefers the medical field is chosen at random, which of the following is closest to the probability that the student is from High School B?

A) 0.111
B) 0.206
C) 0.431
D) 0.541

Practice 18

Based on the table, if a student is chosen at random from High School B or High School C, which of the following is closest to the probability that the student prefers liberal arts?

A) 0.156
B) 0.166
C) 0.313
D) 0.490

Practice 19

Based on the table, if a student is chosen at random, which of the following is closest to the probability that the student is from High School B and prefers engineering or is from High School D and prefers law?

A) 0.035
B) 0.049
C) 0.135
D) 0.719

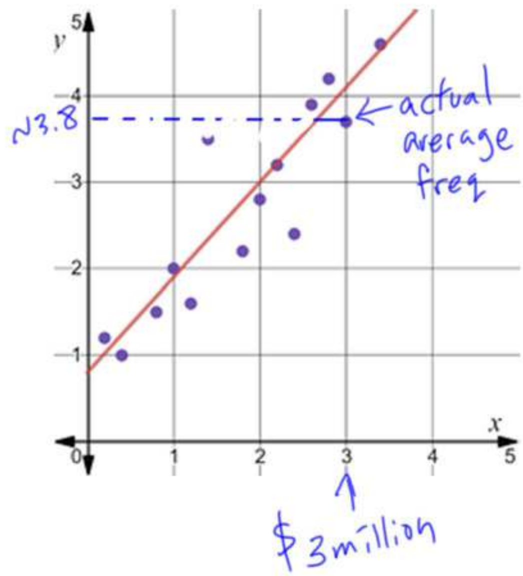
____________∆____________

Answers to Practice Problems

Practice 1: C

Where the survey was given. Since Chloe interviewed students who attended a football game, they are more likely to support renovating the sports field than the overall student population. The sample size is fine. People are expected to not participate, and she can't control it. For a survey, it isn't necessary to survey everyone.

Practice 2: C



Practice 3: 72

Sum of 5 tests is 75

Sum = (Avg)(#)

Sum = (75)(5)

Sum = 375

all 5 tests

We can find the sum of the first 4

$90+80+62+71=303$

first 4

$375-303=\boxed{72}$

5th

Practice 4: 17.5

Calculate the mean (avg.) mass of Mark's seashells

$$\frac{12.4 + 10.2 + 12.0 + 14.2}{4} = \frac{48.4}{4}$$

$$= 12.2$$

Mean of Sandra's is 1 more than Marks, so

Sandra's mean = 13.2

Sum = (Avg)(#)

Sum = (13.2)(4)

Sum = 52.8

Sandra's add up to 52.8

3 of them (13, 10.6, 11.7) add up to 35.3

Missing seashell mass:

$52.8 - 35.3 = \boxed{17.5}$

Practice 5: 83.5

Practice 6: 80

Practice 7: 90

Practice 8: 50,000

Sale Price ($)	Frequency
350,000	6
400,000	3
450,000	7
500,000	8
550,000	+ 12
	36

$\frac{36+1}{2} = \frac{37}{2} = 18.5$

Median is average of 18th + 19th terms

Let's count up from the bottom.

Sale Price ($)	Frequency	
350,000	6	
400,000	3	
450,000	7	
500,000	8	20
550,000	12	12

500,000 is the 13th through 20th terms, so it is the 18th & 19th

The median is 500,000.

The mode is the one that appears the most, the most frequent number. 550,000 has a frequency of 12 which is the most, so 550,000 is the mode.

The mode is 50,000 more than the median.

CADDELL PREP

Practice 9: A

Based on the sample, if there are 5,000 owls total, we can estimate the number of A-owls and B-owls.

Set up a proportion to find the # of A-owls if there are 5,000 owls.

$$\frac{\text{A-owl}}{\text{Total}} \quad \frac{35}{50} = \frac{x}{5000}$$

$$175000 = 50x$$

$$\frac{175000}{50} = \frac{50x}{50}$$

$$3{,}500 = x$$

3,500 A-owls
1,500 B-owls

We can estimate that if there are 5,000 owls, then 3,500 will likely be A-owls. Choice C may appear to be correct because we estimate there to be more A-owls than B-owls. However, there are two things that make it wrong. Choice C definitively says that there are more, which we don't know for sure. Also, choice C states that there are more A-owls than B-owls in the USA, but we only have enough data to make an assumption about the one town.

Practice 10: 50

Based on the table, we can find the total number of Clydesdales since we know there are 216 total horses and 94 Arabian horses.

	Male	Female	Total
Arabian	45		94
Clydesdale		72	+ 122
Total			216

There must be 122 Clydesdales.

Since there are 122 total Clydesdales and 72 of them are female, we can find the number of male Clydesdales.

	Male	Female	Total
Arabian	45		94
Clydesdale	50 +	72 =	122
Total			216

There are 50 male Clydesdales.

Practice: 11

Part 1

	6th Graders	7th Graders	8th Graders	Total
School A	345	395	326	1,066
School B	410	445	432	1,287
Total	755	840	758	2,353

School A & 6th grade

out of

$$\boxed{\frac{345}{1{,}066}}$$

Part 2

	6th Graders	7th Graders	8th Graders	Total
School A	345	395	326	1,066
School B	410	445	432	1,287
Total	755	840	758	2,353

7th & 8th from School B

$445+432=877$

7th or 8th grader

$840+758=1598$

$$\boxed{877/1598}$$

Practice 12: D

64% of the data should fall within one standard deviation of the mean. There are 100 volunteers, so 64% would be 100 volunteers.

The mean is 185 and the standard deviation is 8, so the range would be from 177 (8 less than 185) to 192 (8 more than 185).

Practice 13: 0.57

Day 1: 2.40 in.

Day 7: 2.74 in.

$$\frac{\text{change in height}}{\text{change in time}}$$

$$\frac{2.74 - 2.40}{7 - 1}$$

$$\frac{0.34}{6}$$

0.57 in/day

Practice 14: C

$$\frac{\text{Change in height}}{\text{Change in days}}$$

Plant A

$$\frac{3.51 - 3.25}{5 - 1}$$

$$\frac{0.26}{4}$$

0.065 in/day

Plant B

$$\frac{2.75 - 1.25}{5 - 1}$$

$$\frac{1.5}{4}$$

0.375 in/day

0.375 − 0.065

0.31 in/day

Plant B grew 0.31 in/day faster than Plant A

Practice 15: C

It is likely to help because the participants who used it lost more weight than the participants who didn't. We can't jump to the conclusion in choice A that it's better than other treatments because we aren't given any information about other treatments. We can't choose B because it states that it will work for anyone who takes it, but that's too strong of a statement to make. There can be exceptions. We can't choose choice D because we don't know how much weight-loss it will cause.

Practice 16: B

In the equation $y = \frac{3}{10{,}000}x + 51{,}000$, the number $\frac{3}{10{,}000}$ represents the slope of the trend line. Slope tells us the average rate of change of y over x. In this case y is the average happiness rating and x is the salary.

Practice 17: D

Chosen out of: prefers Medical (5

HS B students who prefer Med.: 29

	Engineering	Liberal Arts	Medical	Law	Business	Total
High School A	252	43	176	102	74	647
High School B	58	75	295	202	54	684
High School C	8	54	2	7	24	95
High School D	24	653	72	35	430	1,214
Total	342	825	545	346	582	2,640

$$\frac{295}{545} = \boxed{0.541}$$

Practice 18: B

Student from HS B or HS C cho

at random.

out of HS B + HS C

$684 + 95 = 779$

	Engineering	Liberal Arts	Medical	Law	Business	Total
High School A	252	43	176	102	74	647
High School B	58	75	295	202	54	684
High School C	8	54	2	7	24	95
High School D	24	653	72	35	430	1,214
Total	342	825	545	346	582	2,640

129

HSB or HSC that prefers Liberal Arts: $75 + 54 = 129$

$$\frac{129}{779} = \boxed{0.166}$$

Practice 19: A

Out of all students: 2640

	Engineering	Liberal Arts	Medical	Law	Business	Total
High School A	252	43	176	102	74	647
High School B	58	75	295	202	54	684
High School C	8	54	2	7	24	95
High School D	24	653	72	35	430	1,214
Total	342	825	545	346	582	2,640

HS B + Engineering: 58

HS D + Law: 35

HS B + Eng or HS D + Law

$58 + 35$

93

$$\frac{93}{2640} = \boxed{0.035}$$